Technical Appendix: A Better Response to Keen-Standish “hill climbers”

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1. Keen “equilibrium”

If all firms follow the strategy suggested by Standish and Keen, and all firm synchronize their movements in output then each firm “gropes" toward

$$q_i^{KS} = \frac{p^{KS} - c_i}{\beta} \frac{\delta_i}{\sum_j \delta_j}$$

as in [1, Section 7.1], resulting in price

$$p^{KS} = \alpha - \beta \sum_i \frac{p^{KS} - c_i}{\beta} \frac{\delta_i}{\sum_j \delta_j} = \alpha - \frac{\sum_i (p^{KS} - c_i)\delta_i}{\sum_i \delta_i} = \alpha - p^{KS} + c_\delta$$

or

$$p^{KS} = \frac{1}{2}\alpha + \frac{1}{2}\frac{\sum_i \delta_i c_i}{\sum_i \delta_i} = \frac{\alpha + c_\delta}{2}$$

where c_δ is the δ-weighted average marginal cost across firms. That is,

$$p^{KS} - c_i = \frac{1}{2}[\alpha + c_\delta - 2c_i] = \frac{1}{2}[(\alpha - c_i) - (c_i - c_\delta)]$$

resulting in profits

$$\pi_i^{KS} = \frac{1}{4\beta}\frac{\delta_i}{\sum_j \delta_j}[(\alpha - c_i) - (c_i - c_\delta)]^2 \tag{1}$$

Thus, the profits of firm i increase with the firm’s δ-share

$$\theta_i \equiv \frac{\delta_i}{\sum_j \delta_j}$$

but if the firm’s marginal cost is lower than that of its competitors, a greater δ lowers the average marginal cost and therefore the supply price. Defining relative marginal cost

$$\xi_i \equiv \frac{c_i}{c_\delta^r}$$

We may rewrite (1) as

$$\pi_i^{KS} = \frac{\theta_i}{4\beta}\left[(\alpha - c_i) - (1-\theta_i)(1-\frac{1}{\xi_i})c_i\right]^2$$

Figures 1 and 2 show the profits of a firm in Keen “equilibrium”.

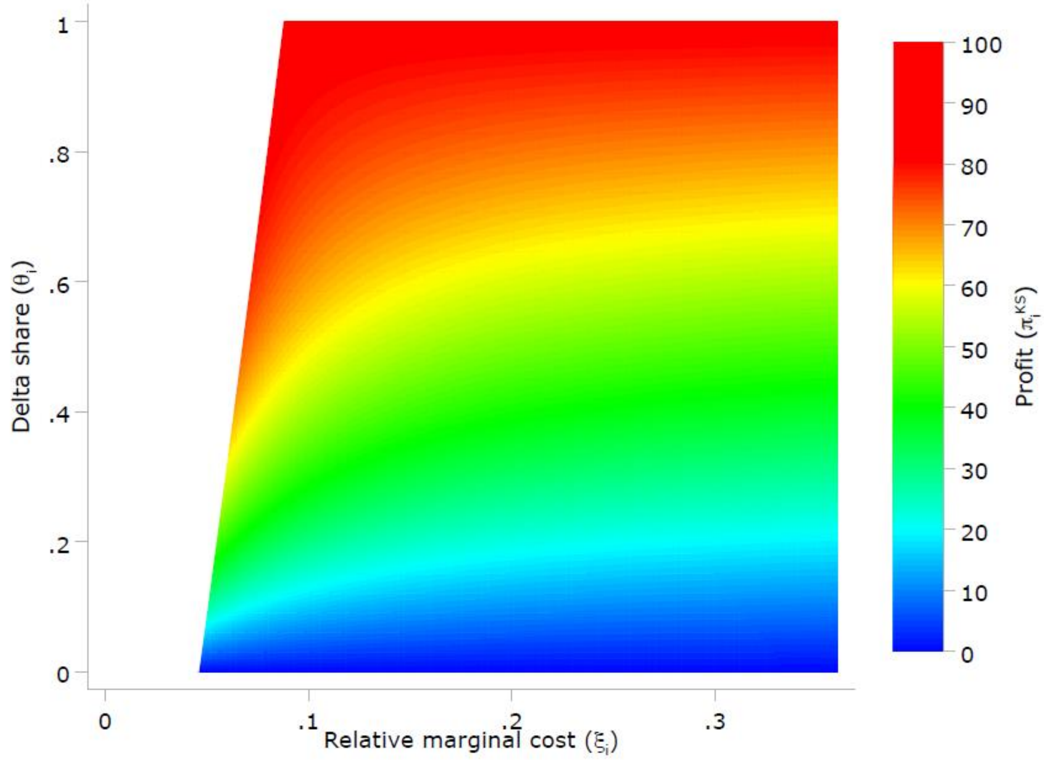

Figure 1: Profits of a firm with lower marginal cost

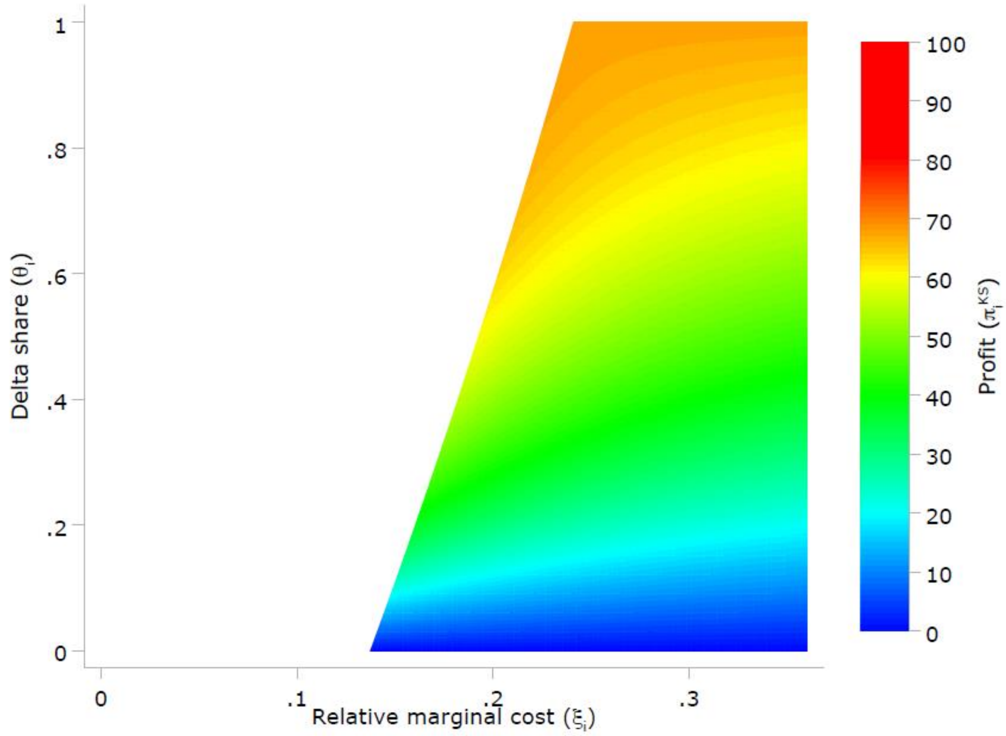

Figure 2: Profits of a firm with higher marginal cost

As is apparent from these figures, for any ξ_i a larger θ_i means higher profits. However, if the firm's marginal cost is sufficiently low relative to its competitors, there is a limit to how much of the market it may capture before guaranteeing that at least one other firm will be unprofitable. If all firms have positive profits then the residual average marginal cost must be less than the price, or

$$2c_\delta^r < 2p^{KS} = \alpha + c_\delta = \alpha + \theta_i \xi_i c_\delta^r + (1-\theta_i) c_\delta^r$$

That is,

$$(1-\xi_i)\theta_i < \frac{\alpha}{c_\delta^r} - 1$$

If ξ_i is sufficiently small the market price must be too low for all firms to operate profitably. At least one high-cost firm must drop out, lowering the average marginal cost of its competitors c_δ^r and increasing ξ_i. Thus, firm i cannot operate in the region to the left of the regions indicated in Figures 1 and 2.

Note, however, that everywhere else, the firm has higher profits the greater the firm's δ-share. That is, a firm following the strategy of Standish and Keen should not satisfy itself with whatever δ the authors assign it. Rather, the firms prefer a δ very large in comparison to its competitors.

2. One firm changes strategy

If all other firms follow the strategy described by Standish and Keen, but firm i instead follows the strategy described in [1, Section 6.4] then firm i produces

$$q_i^S = \frac{1}{2\beta}[(\alpha - c_i) - (c_i - c_\delta^r)]$$

resulting in a price such that

$$p - c_i = \frac{\alpha + c_\delta^r}{2} - \frac{1}{4}[(\alpha - c_i) - (c_i - c_\delta^r)] - c_i = \frac{1}{4}[(\alpha - c_i) - (c_i - c_\delta^r)]$$

and profits

$$\pi_i^S = \frac{1}{8\beta}[(\alpha - c_i) - (c_i - c_\delta^r)]^2 = \frac{1}{8\beta}\left[(\alpha - c_i) - \left(1 - \frac{1}{\xi_i}\right)c_i\right]^2 \qquad (2)$$

3. Choice of response to Standish-Keen competitors

If we persist in assuming firms are stuck with their assigned δ and $c_i = c_\delta$, then

$$\pi_i^{KS} = 2\theta_i \pi_i^S$$

so any firm with less than half the δ share would be better off changing strategies. In general, a firm must have a considerable δ-share to justify sticking with the advice of Standish and Keen. For a given θ_i, the strategies break even when

$$[(\alpha - c_i) - \left(1 - \frac{1}{\xi_i}\right)c_i]^2 = 2\theta_i[(\alpha - c_i) - (1-\theta_i)\left(1 - \frac{1}{\xi_i}\right)c_i]^2$$

or

$$\left(1 - \frac{1}{\xi_i}\right)c_i = \frac{1 - \sqrt{2\theta_i}}{1 - (1-\theta_i)\sqrt{2\theta_i}}(\alpha - c_i)$$

As Figure 3 shows, this line divides the ξ-θ map into two regions: the "Keen" region where it is more profitable for the firm to follow the strategy of Standish and Keen, and the "Stackelberg" region, where it is more profitable to operate at a steady level of output and let the rest of the competition settle into a "Keen equilibrium".

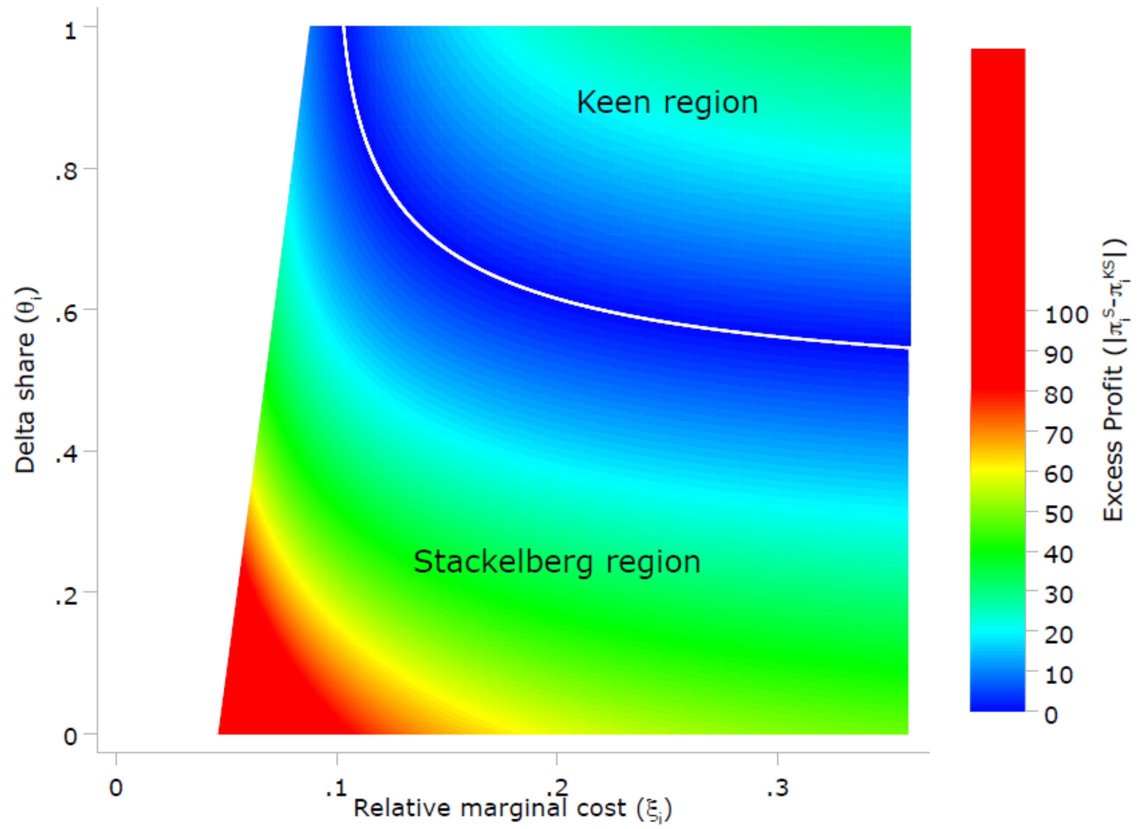

Figure 3: Absolute difference in profits between strategies

The Standish-Keen strategy requires a very large δ share indeed.

4. Analysis of the Standish-Keen simulations

Let us conclude with the parametrization presented in [2]. There,

$$P(Q^s) = 11 - \frac{1}{3}Q^s$$

and all firm marginal costs $c \in [0.5; 1.5)$. For a firm with the lowest possible marginal cost, $\xi \in (\frac{1}{3}, 1]$; for a firm with the highest, $\xi \in [1, 3)$. In Figures 4 and 5 we see the ξ-θ maps for the lowest and highest marginal cost firms, respectively.

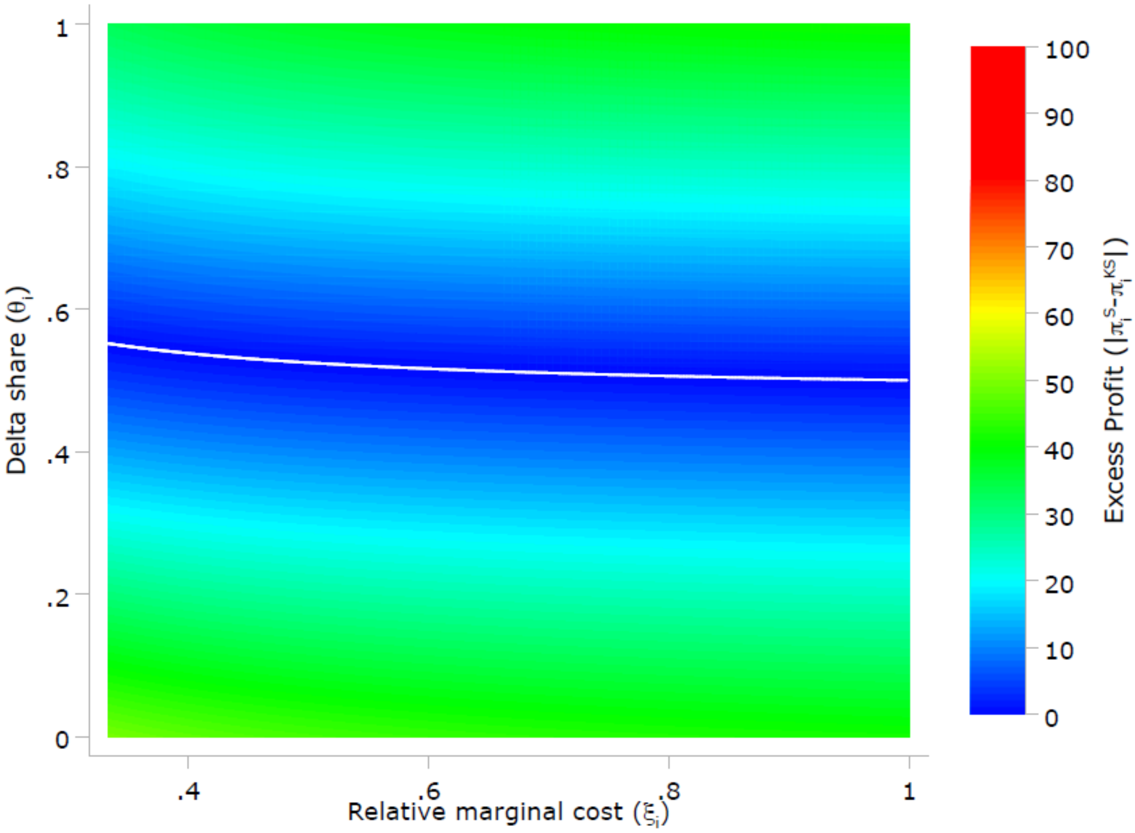

Figure 4: ξ- θ map for $c = 0.5$

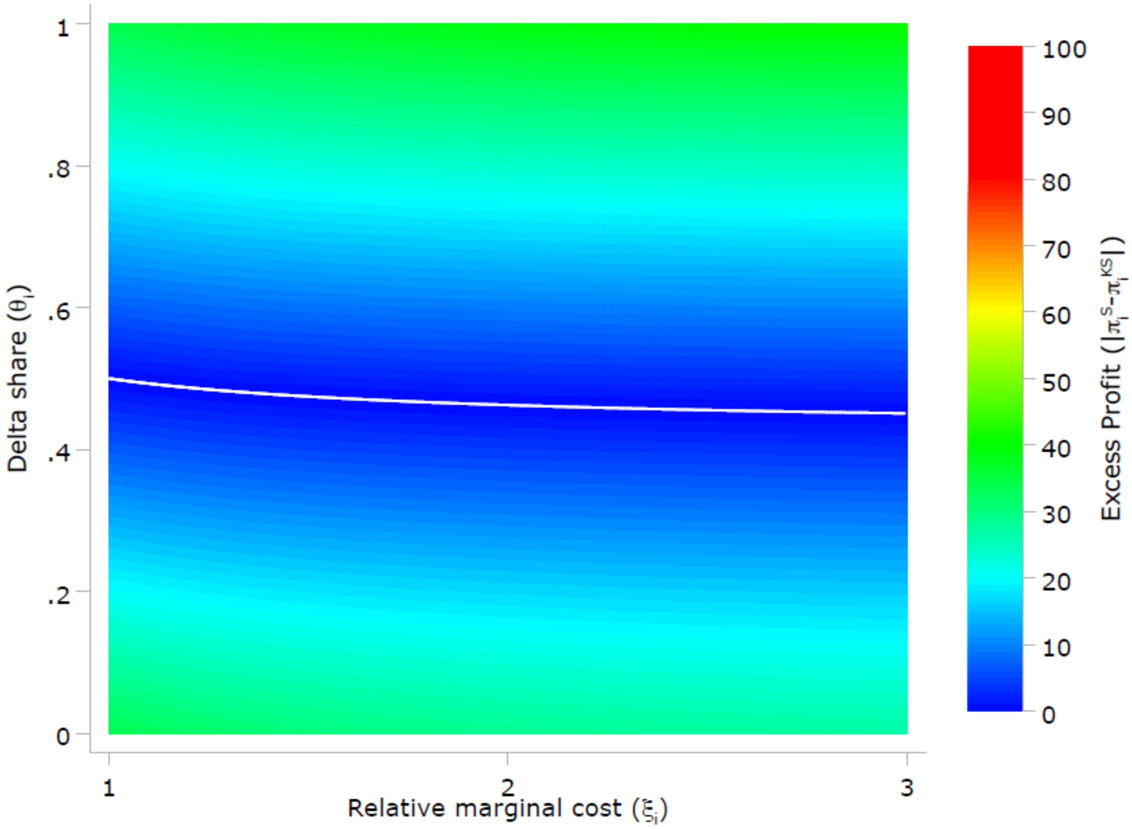

Figure 5: ξ- θ map for $c = 1.5$

Clearly, in the simulations of Standish and Keen, nearly every firm would profit from a change in strategy. With 1,000 firms in competition, for a firm to see even $\theta > 1/10$ this would require the firm to have a step size 111 times larger than the average of its competitors. The authors' chosen parameterization of the half-normal distribution of δ is not nearly wide enough to make such an event likely. Regardless, at most two firms would fail to benefit from a change in strategy. The Keen result is not a competitive equilibrium.

References

[1] David Rosnick. 'Toward an Understanding of Keen and Standish's Theory of the Firm: A Comment'. *World Economic Review* (5) pp. 107-123, 2015.

[2] Russell K. Standish and Stephen L. Keen. 'Rationality in the theory of the firm'. *World Economic Review* (5) pp. 101-106, 2015.