

Response to David Rosnick’s “Toward an Understanding of Keen and Standish’s Theory of the Firm: A Comment”

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Rosnick makes a lot of detailed points, but has failed to understand exactly what theory we are criticising. He organises his response according to three rather interwoven threads, to which we will only respond to the most important issues, rather than make a detailed point-by-point response.

We will start with the issue that Rosnick completely failed to discuss (and which we therefore label as Thread 0): that the so-called profit-maximizing formula for an individual firm – of equating marginal cost and marginal revenue – provably does not maximize profits in any industry structure apart from monopoly.

Thread 0

Standard Neoclassical pedagogy teaches that, regardless of market structure, an individual firm in an industry will maximize its profits by equating its marginal revenue (the derivative of its revenue to its output) to its marginal cost (the derivative of its total cost of production with respect to its output):

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial (P(Q) \cdot q_i)}{\partial q_i} - \frac{\partial TC_i(q_i)}{\partial q_i}. \quad (1)$$

However, given the standard Neoclassical assumptions of falling marginal revenue and rising marginal cost [1, 2]¹ the profit maximum for any individual firm is found, not by setting what is in fact the *partial derivative* of its profit to zero, but by setting the *total derivative* of its profit to zero:

$$\frac{d\pi_i}{dQ} = \sum_j \frac{\partial \pi_i}{\partial q_j} \frac{dq_j}{dQ} = 0. \quad (2)$$

Our paper in this issue generalizes our earlier analysis to cover asymmetric firm response, but in the interests of illustrating the crucial point that Rosnick ignores, we provide a simple comparison of the standard “Neo-classical profit maximization” formula and the actual profit maximization formula in the case of n identical firms in an industry.

Consider a linear demand curve $P(Q) = a - bQ$ and an industry with n identical firms, where each firm has the identical total cost function $TC(q) = k + cq + dq^2/2$. Then the total revenue for an individual firm will be $TR(q) = P(Q)q = aq - bnq^2$ and profit will be:

$$\pi(q) = a \cdot q - b \cdot n \cdot q^2 - \left(k + c \cdot q + \frac{1}{2} \cdot d \cdot q^2 \right) \quad (3)$$

¹ The assumption of rising marginal cost has been found to be false by all empirical studies of firm cost structures. See [2] for a survey and [1, Chapter 4] for the most recent research.

Applying Stigler's rule [3, p. 8 footnote 31] that $\partial P / \partial q_i = dP/dQ$, we can derive marginal revenue for the individual identical firm as $MR(q) = P + qdP/dQ = a - b(n + 1)q$, while marginal cost is $MC(q) = c + dq$. The output level q_N that results from following the Neoclassical rule of equating marginal revenue and marginal cost is:

$$q_N = \frac{a - c}{(n + 1)b + d} \quad (4)$$

In the case of n identical firms, the actual ("Keen") profit maximization formula can be written as:

$$MR(q) - MC(q) = \frac{n-1}{n}(P(nq) - MC(q)) \quad (5)$$

The solution q_K to this formula is:

$$q_K = \frac{a - c}{2bn + d} \quad (6)$$

The profit level that results from the q_K level of output exceeds that from the q_N level for $n > 1$:

$$\pi(q_K) - \pi(q_N) = \frac{b^2(a - c)^2(n - 1)^2}{2(2bn + d)(b + d + bn)^2} > 0 \forall n > 1. \quad (7)$$

For a numerical illustration, consider an example with parameter values $a = 100$, $b = 10^{-6}$, $c = 20$, $d = 10^{-5}$, $k = 10000$ and 100 firms ($n = 100$). In this case the profit per firm from the q_K level of output is 1.523×10^7 versus 3.107×10^6 for the q_N level of output. Equating marginal cost and marginal revenue is therefore not a profit-maximizing strategy for the individual firm – though it may be the result of competitive interactions between firms. Our paper considers the stability of these interactive strategies.

Thread 1

The theory we're critiquing is the Marshall theory of the firm, which like the Cournot oligopolists, consists of price-taking agents operating in a clearing market with the market prices determined by a given function of total production Q . There is no separate supply and demand Q s, or if there are, they are identical, always. The Marshall model also supposes perfect competition, namely that the individual firms have no influence on market price, or $\partial P / \partial q_i = 0$, which as Rosnick agrees with us, is simply incompatible – a logical fallacy.

Thread 2

Given that the construction of the Marshall model (minus the perfect competition condition imposed by fiat) is also the same as classical Cournot analysis, Rosnick turns his attention to the Cournot game. However, the Cournot game is defined as a single shot game, where firms must make decisions on their output, accept the profit received and that is the end of the game. Again, the model we're critiquing is not that, as firms iteratively set production values and receive profits, thus allowing agents to gain information about each other inductively. Just as semi-cooperative strategies beat totally defecting strategies in iterated prisoners dilemma, the same happens in the Marshall model.

It is true that we haven't specified dynamical equations for the Marshall model, as the model is not sufficiently detailed to specify it completely, but suppose the dynamical equations are:

$$\frac{d\mathbf{q}}{dt} = \mathbf{F}(\boldsymbol{\pi}(\mathbf{q})), \quad (8)$$

where $\mathbf{q} = (q_1, q_2, \dots, q_n)$, $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$ and generally we denote vector values quantities in bold face. \mathbf{F} has to be a function of the profit vector $\boldsymbol{\pi}$, as the agents' behaviour is entirely determined by their profit seeking rationality.

Eq (8) only has equilibria where the derivative $D_{\mathbf{q}}F = 0$, or equivalently where $\partial F_i / \partial q_j = 0, \forall i, j$.

The chain rule is

$$D_{\mathbf{q}}F = D_{\boldsymbol{\pi}}\mathbf{F} \cdot D_{\mathbf{q}}\boldsymbol{\pi}, \quad (9)$$

where \cdot is the usual matrix multiplication. Assuming $D_{\boldsymbol{\pi}}\mathbf{F}$ is full rank, then the only equilibria occur when $\partial \pi_i / \partial q_j = 0, \forall i, j$ which, as we show in our paper, is only possible for the trivial solution $Q = 0$. Indeed the simplest scenario is for $D_{\mathbf{q}}\mathbf{F}$ to be proportional to $D_{\mathbf{q}}\boldsymbol{\pi}$, in which case each firm is equally striving to improve its own profit independently of the others.

The Cournot outcome has an unstable manifold, where overall production is declining leading to rising profits for all. The only way to prevent the system from taking that path is to explicitly inhibit it in some way – it might be said that prevention of collusion is sufficient to prohibit this from occurring, but the simple agent model we provide is a counter example, where non-collusive firms nevertheless end up at monopoly-like market levels.

But if an actual mathematical equilibrium does not exist, there is still the possibility of a *macroeconomic equilibrium*, where the individual firm's profits do not change as total production Q changes, ie

$$\frac{d\pi_i}{dQ} = 0. \quad (10)$$

This is the origin of the Keen formula. Expanding (10) as

$$\frac{d\pi_i}{dQ} = \sum_j \frac{\partial \pi_i}{\partial q_j} \frac{dq_j}{dQ} = 0, \quad (11)$$

and substituting into a linearised version of (8) around a Keen outcome \mathbf{q}^K , we get

$$\begin{aligned} \frac{d\mathbf{q}}{dt} &= \sum_j \frac{\partial \pi_i}{\partial q_j} (q_j - q_j^K) \quad (12) \\ &= 0 \text{ if } Q - Q^K \neq 0. \end{aligned}$$

The condition describes not an equilibrium point, but rather an equilibrium manifold of constant total market production, which is stabilised by the agents ensuring that if any agent were to cause the system to stray from this manifold, then all agents will follow suit, causing that agent to not enjoy its advantage for long. Any rational agent will then return to the fold.

Thread 3

This focuses on the simple agent model we use to explore this issue. Rosnick compares the model to agents dancing in a wagon careering downhill. Whilst poetic, it is not a useful metaphor, chiefly because with the wagon, the system's dynamics are largely determined by the trajectory of the wagon, not the individual agents, whereas with our model, the dynamic behaviour is completely determined by the actions of the agents.

In §6.3, Rosnick introduces a variant of our agents where the decision is made to reverse the usual decision with small probability that increases the closer to equilibrium the system is. Undoubtedly, this could

be done so as to benefit the agent concerned, to the detriment of the other agents in the system, however this action cannot be considered rational. Rational agents always choose the optimum action — the only time rational agents are permitted to act stochastically is when multiple equally valued courses of action are available.

Rational agents should be aware of the possibility that their competitors may not be similarly rational, and take suitable action when they find their market share being stolen from them. Admittedly, the agents specified in our paper are not sophisticated enough to do this, but this was a deliberate choice, since we wanted to show that agents following a simple iterative and non-collusive algorithm would choose output levels that clustered around the true profit-maximizing level of output, and not the Cournot level.

References

- [1] Alan S. Blinder, Elie R. Canetti, David E. Lebow, and Jeremy B. Rudd. *Asking About Prices: A New Approach to Understanding Price Stickiness*. Russell Sage Foundation, New York, 1998.
- [2] F.S. Lee. *Post Keynesian price theory*. Cambridge University Press, Cambridge, 1998.
- [3] George J. Stigler. 'Perfect competition, historically contemplated.' *The Journal of Political Economy*, 65:1–17, 1957.